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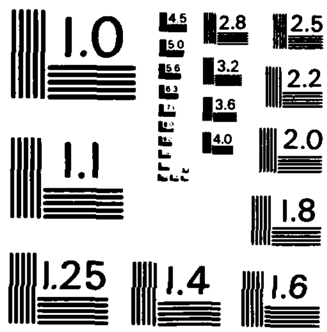

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OPTIMUM ALLOCATION IN MULTISTATE SYSTEMS, WITH APPLICATIONS IN RELIABILITY

by

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ABSTRACT

In this paper we present some results in the optimal allocation of multistate components in a parallel-series system. In addition, we show how these results may be used to obtain fruitful applications in reliability theory. Our basic mathematical tools are majorization and Schur functions; the theorems obtained and methods used are related to those of "Optimal Allocation of Components in Parallel-Series and Series-Parallel Systems," El-Neweihi, E., Proschan, F., and Sethuraman, J., Report (1984). The present paper and the reference just above are the only papers exploiting the elegance and power of majorization and Schur functions to solve optimal allocation problems in reliability, as far as we know.

1. Introduction.

In this paper we present some results in the optimal allocation of multi-state components in a parallel-series system. In addition, we show how these results may be used to obtain fruitful applications in reliability theory. Our basic mathematical tools are majorization and Schur functions; the theorems obtained and methods used are related to those of [2]. The present paper and reference [2] are the only papers exploiting the elegance and power of majorization and Schur functions to solve optimal allocation problems in reliability, as far as we know.

Preliminaries.

For vector $\underline{x} = (x_1, \ldots, x_n)$, let $x_{[1]} \geq \ldots \geq x_{[n]}$ denote the *decreasing rearrangement* of the coordinates of $\underline{x}$. We say vector $\underline{x}$ *majorizes* vector $\underline{y}$ if

$$\sum_1^k x_{[i]} \geq \sum_1^k y_{[i]} \quad \text{for } k = 1, \ldots, n-1$$

and

$$\sum_1^n x_{[i]} = \sum_1^n y_{[i]}.$$

We say a function f: $R^n \to R$ is *Schur-convex*(*Schur-concave*) if $f(\underline{x}) \geq f(\underline{y})\,(f(\underline{x}) \leq f(\underline{y}))$ whenever $\underline{x} \overset{m}{>} \underline{y}$.

Examples of Schur-convex (Schur-concave) functions are $\sum_{i=1}^n f_1(x_i)$ and $\prod_{i=1}^n f_2(x_i)$, where f_1: $R \to R$ is convex (concave) and f_2: $R \to R$ is log-convex (log-concave).

A random variable X is said to be *stochastically larger* than a random variable Y if $P(X > x) \geq P(Y > x)$ for all real x; we write $X \overset{st}{>} Y$. For vectors, $\underline{X} \overset{st}{>} \underline{Y}$ if $f(\underline{X}) \overset{st}{>} f(\underline{Y})$ for every increasing function f.

2. Optimum Allocation of Multistate Components.

We review a general optimal allocation result for multistate systems recently obtained in [2]. We then describe in some detail two models in which this general result can be used.

The following theorem is basic in [2]; the proof depends on the powerful tools of majorization and Schur functions. We present an outline of this proof; the interested reader may consult [2] for further details.

2.1. Theorem. Let $P_1, \ldots, P_k$ be the disjoint min path sets of a parallel-series system having path lengths $n_1, \ldots, n_k$. Without loss of generality, assume that $n_1 \le \ldots \le n_k$. Suppose that there are $n = n_1 + n_2 + \ldots + n_k$ independent components with reliabilities $p_1, \ldots, p_n$ (at time t_o, say) to be allocated among the path sets. Then the reliability of the system (at time t_o) is maximized when the n_1 most reliable components are allocated to P_1, the n_2 next most reliable components are allocated to P_2, ..., and finally, the n_k least reliable components are allocated to P_k.

Proof. Let $x_i = \log \prod_{j \in P_i} p_j$, $i = 1, \ldots, k$. The allocation described in the hypothesis maximizes $\underline{x} = (x_1, \ldots, x_k)$ in the sense of majorization. The reliability function of the system can now be viewed as a Schur-convex function of $\underline{x}$ and the result thus follows. ||

For a more detailed proof, see [2].

Note that the optimal allocation in Theorem 2.1 does not depend on the actual values of the reliabilities $p_1, \ldots, p_n$, but only on their ordering.

This result can be extended to cover the case of multistate components and systems. Consider a parallel-series system as described in Theorem 2.1, except that now each component has a common state space $S \subseteq [0,\infty)$; common choices for S are $\{0, 1, \ldots, M\}$ and the unit interval $[0,1]$. Let $X_1(t), \ldots, X_n(t)$ denote the states of the components 1, ..., n and $X(t)$ denote the state of the system at time t, $t \geq 0$. Suppose that

$$X_1(t) \overset{st}{<} X_2(t) \overset{st}{<} \cdots \overset{st}{<} X_n(t) \tag{2.1}$$

for each $t \geq 0$. Theorem 2.2 just below describes the best allocation of components to path sets to maximize the system state $X(t)$ in the stochastic state uniformly in t.

2.2. Theorem. Consider a parallel-series system as described above with component states satisfying (2.1). To maximize the system state stochastically uniformly in t, allocate components as follows:

The best n_1 components (those with stochastically largest states) to P_1, the next best n_2 components to P_2, ..., and finally the n_k worst components to P_k.

Proof. Fix a time $t \geq 0$ and a state $\alpha \geq 0$. Then $P(X(t) \geq \alpha) = h(p_1^\alpha(t), \ldots, p_n^\alpha(t))$, where $p_i^\alpha(t) = P(X_i(t) \geq \alpha)$ and $h(p_1, \ldots, p_n)$ is the reliability function of a parallel-series system. The result now follows from Theorem 2.1 and Condition (2.1). ||

3. Applications in Reliability.

In this section we describe two models to which Theorem 2.2 can be applied. *In both models the state space is* $\{0, 1, \ldots, M\}$.

3.1. Models. Consider n independent binary components. Suppose that each component is supported by $M-1$ functioning spares that do not deteriorate until put into use. We say that "position" or "socket" i is in state M when the original component of type i is still functioning and none of the $M-1$ spares have been used; the position is in state $M-1$ when the original component has failed and has been replaced by a spare, leaving $M-2$ spares available for replacement; ...; and finally, the position is in state 0 when the last of its spares has failed.

Next let T_j^i be the life length of the j^{th} spare for component type i, $1 \le i \le n$, $1 \le j \le m$. Note that each original component in operation at time 0 is viewed as a member of its spares kit. Assume that for each i, $T_1^i, \ldots, T_M^i$ are independently and exponentially distributed with parameters $\lambda_1^i, \ldots, \lambda_M^i$ respectively. Let $\lambda_{(1)}^i \le \lambda_{(2)}^i \le \ldots \le \lambda_{(M)}^i$ be a rearrangement of $\lambda_1^i, \ldots, \lambda_M^i, i = 1, \ldots, n$. Assume that $\lambda_{(\ell)}^1 \ge \lambda_{(\ell)}^2 \ge \ldots \ge \lambda_{(\ell)}^n$ for $\ell = 1, \ldots, M$.

Note that for each pair (i,t), the distribution of $X_i(t)$ depends on the order in which we have been using the spares. However, a little reflection shows that the following order maximizes $X_i(t)$ stochastically, uniformly in t for each i: Start with the spare whose parameter is $\lambda_{(1)}^i$. Upon its failure, replace it with the spare whose parameter is $\lambda_{(2)}^i$, ..., and finally use the spare whose parameter is $\lambda_M^{(i)}$. Let $X_i^*(t)$ be the random state of component i corresponding to the above order. Then clearly $X_1^*(t) \overset{st}{<} X_2^*(t) \overset{st}{<} \ldots \overset{st}{<} X_n^*(t)$ for each t. The allocation described in Theorem 2.2 maximizes stochastically the system state uniformly in t.

Thus, employing the order described above for using the spares corresponding to each socket, together with the optimal allocation of Theorem 2.2 yields the best stochastic performance of a parallel-series system formed from the n multi-state components. ||

3.2. Remark. In the example given in [2], $\lambda_1^i = \lambda_2^i = \ldots = \lambda_M^i = \lambda_i$, $i = 1, \ldots, n$. Thus, in this case, the order in which the spares are used is immaterial.

3.3. Remark. In Model 3.1, it is assumed that for each i, $T_1^i, \ldots, T_M^i$ have exponential distributions with parameters satisfying certain conditions. Other distributions can be used provided the following conditions hold: 1) For each i, the random variables $T_1^i, \ldots, T_M^i$ are stochastically ordered. 2) For $i < j$, the smallest stochastically among $T_1^i, \ldots, T_M^i$ is stochastically less than the smallest among $T_1^j, \ldots, T_M^j$, the second smallest among $T_1^i, \ldots, T_M^i$ is stochastically less than the second smallest among $T_1^j, \ldots, T_M^j$, ..., and finally the largest stochastically among $T_1^i, \ldots, T_M^i$ is stochastically less than the largest stochastically among $T_1^j, \ldots, T_M^j$.

3.2. Model.

Consider $n \cdot M$ independent binary components forming n parallel systems $S_1, \ldots, S_n$ of size M each. We now view each S_i as a single multistate system of components with $M+1$ states 0, 1, ..., M, as follows: When all the M binary components in system S_i are functioning, then the socket corresponding to multistate component i is in state M. When the first binary component in system S_i fails, socket i is now in state $M-1$, and so on, until the last binary component in system S_i fails, and socket i is now in state 0, $i = 1, \ldots, n$. Let $\underline{T}^i = (T_1^i, \ldots, T_M^i)$ be the random vector representing the joint lifelengths of the M binary components in system S_i, $i = 1, \ldots, n$. Suppose that $\underline{T}^1 \overset{st}{<} \underline{T}^2 \overset{st}{<} \cdots \overset{st}{<} \underline{T}^n$. Let $X_1(t), \ldots, X_n(t)$ be the states of components 1, ..., *n* at time *t*, $t \geq 0$. Then $P(X_i(t) > j) = P(T^i_{(M-j)} > t)$ for each $j = 0, \ldots, M-1$; $i = 1, \ldots, n$, and $t \geq 0$, where $T^i_{(\ell)}$ is the ℓ-th order statistic among $T_1^i, \ldots, T_M^i$. Clearly $X_1(t) \overset{st}{<} X_2(t) \overset{st}{<} \cdots \overset{st}{<} X_n(t)$ for each $t \geq 0$. Theorem 2.1 can now be used to provide us with the optimal allocation of the n multistate components to the min path sets of a parallel-series system of n components. ||

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[2] Optimal Allocation of Components in Parallel-Series and Series-Parallel Systems. El-Neweihi, E., Proschan, F., and Sethuraman, J. Report (1984), submitted for publication.

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